FOLIA 231

Annales Universitatis Paedagogicae Cracoviensis

Studia ad Didacticam Mathematicae Pertinentia 9(2017)

ISSN 2080-9751
DOI 10.24917/20809751.9.12

Bożena Rożek

On the monograph "Mathematical Transgressions 2015"*

For the second time, mathematics educators from the Institute of Mathematics of the Pedagogical University of Kraków (Poland) have organized an interdisciplinary scientific conference on various mathematical issues bordering different scientific fields, entitled the 2^{nd} Interdisciplinary Scientific Conference "Mathematical Transgressions". It was held on March 15–19, 2015.

The idea of such a conference emerged among Polish mathematics educators who recognized an urgent contemporary demand for integrating scientists representing seemingly distant fields of knowledge into the successful education of creative students, particularly those who are mathematically fit. The participants of this conference were scientists representing various fields, e.g. mathematics, mathematics education, pedagogy, psychology and philosophy.

Selected papers have been published in the monograph (Błaszczyk, Pieronkiewicz, 2018). In the introduction, the editors point out that:

> "During the last decades of the 20th century, we have entered the Digital Era. The Third Technological Revolution has an enormous socioeconomic impact. It affects modern science, including mathematics... We should agree that mathematics is no longer the Queen of the Sciences; while it is still believed to be the basis of modern education, its role needs to be re-defined. It is necessary to address this challenge. The first step in this direction consists of the adoption of a new perspective. The Latin word "transgression" means an act teat goes beyond generally accepted boundaries. This monograph draws together papers written by mathematicians, educators, pedagogues, psychologists, and philosophers. Their aim is to identify a new role of mathematics and mathematics education in the modern world" (Błaszczyk, Pieronkiewicz, 2018, p. 5).

The subject area of the monograph varies, as shown in the contents below, while in the following part, three selected articles have been abstracted, and the paper by E. Gruszczyk-Kolczyńska has been summarized more closely.

*On the monograph "Mathematical Transgressions 2015"

The book consists of three chapters which include the following papers:

Philosophy and Psychology of Mathematics

- Paul Ernest, Challenging three myths about mathematics: Recognising the social responsibility of mathematics
- Shlomo Vinner, Scientific thinking versus religious thinking from a view point of a Hsecular science educator
- Bernard Sarrazy, Contrat, transgressions et creation. Une tentative de clarifier les paradoxes de la relation didactique dans l'enseignement des mathematiques en utilisant une approche didactique et anthropologique
- Jérôme Proulx, Looking at students' mathematics: from a deficit view on mathematical knowledge toward possibilities of mathematical actions
- Barbara Pieronkiewicz, Affective transgression – a new perspective on the problem of low achievement in learning mathematics
- Manuela Moscucci, About the relation between relationships and teaching and learning mathematics

Research on Mathematics Education

- Erich Ch. Wittmann, Structure-genetic Didactical Analyses. Empirical Research "of the first kind"
- Jeppe Skott, The Goldilocks principle revisited: understanding and supporting teachers' proficiency with reasoning and proof
- Mette Andresen, Glimpses of students' mathematical creativity, which occurred during a tudy of students' strategies for problem solving in upper secondary mathematics classes
- Jarmila Novotná, Pets Eisenmann, Jiří Přibyl, The effects of heuristic strategies on solving of problems in mathematics
- Aneta Sondore, Elfrıda Krastiņa, Comprehension of elements of combinatorics in real-life situations among primary school students
- Hannes Stoppel, Development of students' beliefs in mathematical understanding in relationship to mathematics and its application
- Edyta Gruszczyk-Kolczyńska, Mathematically gifted children: research results, analysis, conclusions
- Ralf Benölken, Research results on mathematical talent, gender and motivation
- Eva Nováková, Prediction and self-evaluation as a part of the process of solving non-standard mathematical task
- Jacobus G. (Kobus) Maree, Career construction in the mathematics classroom: using an integrated, qualitative + quantitative approach to enhance learners' sense of self
- András Ambrus, Applying cognitive load theory in mathematics education

Mathematics, Arts and other Science

- Jerzy Pogonowski, Paradox resolution as a didactic tool
- Urszula Foryś, Interdisciplinary character of mathematics: biomathematical perspective
- Maria Gokieli, Marcin Szpak, Visualization and Experiment in (School) Mathematics
- Agata Hoffmann, The use of Stanisław Drożdż's works as teaching aids in mathematics
- Jakub Jernajczyk, Bartłomiej Skowron, Circle and sphere – geometrical speculations in philosophy
- Paola Vighi, Abstract paintings, objects, and actions: how promote geometrical understanding

The opening chapter of the book, *Challenging three myths about mathematics: Recognising the social responsibility of mathematics*, written by Paul Ernest (2018), addresses several misleading, yet commonly held views on mathematics.

The Author claims that while mathematics is usually seen as a unique and unified subject, there is no single, isolated object that it refers to. To dismiss this absolutist perception of mathematics, the Author unfolds a metaphor of mathematics as a dynamic, growing city with a plethora of different kinds of buildings (e.g., skyscrapers – representing the formal theories in mathematics, academic centres – meaning the various branches of research within the field, favelas – for informal mathematics, schools – symbolising mathematics education). This city model represents different types of knowledge as well as practices that make up mathematics, termed with a single word, but covering a maze of complexities.

The second myth questioned in the paper is that of mathematics as a value and ethics-free domain. In order to address this controversial issue, the Author refers to and extends his previous considerations (e.g., Ernest, 1998, 2007, 2015) on the epistemological and epistemic, ontological and ontic, as well as aesthetic and ethical values embedded in mathematical knowledge. Among the many different kinds of values to be found in and supported by mathematics, the Author lists, for example, openness, fairness, and democracy. In recent decades, there have been many attempts made toward defining the core of mathematics. For instance, Mason, Graham, and Johnson-Wilder (2005) find it in expressing generality, Holyes and Jones (1998) see it in proof, and Franklin (2017) in the distinction between the discrete and the continuous. Perhaps only a few statements formulated in order to capture the essence of mathematics will go down in history. Undoubtedly, one of such legendary statements comes from Cockcroft (1982), to whom problem solving is what lies at the heart of mathematics. However, Ernest's utterance, modelled on that of Cockroft, stating that "conversation lies at the heart of mathematics" seems to be its worthy successor, emphasizing the human foundations on mathematics.

The third commonly held view on mathematics challenged in the paper is that mathematics is undoubtedly a force for good. While, unarguably, mathematics has a positive impact on various dimensions of human life, the Author directs our attention to the harm that mathematics can cause when applied and taught in an incorrect manner. The detrimental aspects of mathematics are, for instance, the masculine image of mathematics, which negatively impacts women, and the impact mathematics has on the attitudes and self-esteem of minorities. In that sense, it can be said that mathematics is a double-edged sword (Pieronkiewicz, Szczygieł, 2018).

In his paper entitled *Structure-genetic Didactical Analyses. Empirical Research 'of the first kind'*, E. eh. Wittmann (2018) strongly criticizes the observable shift in the direction of mathematics education research from the theories emerging from and based on the subject matter of mathematics as a discipline towards imported general theories of learning and teaching (see also: Wittmann, 1995). The Author advocates for bridging back more appropriate proportions and relevant use of these two approaches as well as grounding research within the field of mathematics education.

Wittmann highlights the importance of n research method called the 'structure-genetic didactical analysis' in mathematics education which is an extension of the traditional "subject matter didactics." This method includes not only logical analysis of the subject matter, but also takes into account the genesis of knowledge over grades and individual learning processes. The term conceived for this research orientation shows an explicit link with the work of Piaget on genetic epistemology. In the Author's words:

> "The structure-genetic didactical analysis is an empirical method. Because of its nativeness it may be well considered as empirical research of "the first kind". The usual empirical studies are then empirical research of the "second kind". The assertion that only empirical studies of the second kind would provide "evidence-based models" for teaching and learning is untenable" (Wittmann, 2018, p. 144).

The following arguments justify the use and confirm the importance of structure-genetic didactical analysis in mathematics education:

1. The analysis originates from mathematical practice.
2. It supports an active relationship with mathematics as a living discipline.
3. This research method is constructive and thus essential if the aim is to provide students with *substantial learning environments* (see: e.g., Wittmann, 2001, 2005; Krauthausen, Scherer, 2010).
4. Such an analysis provides teachers with natural guidelines and encompasses implicit theories of teaching and learning of mathematics by revealing didactical moments which are already there ("freezed") in the subject.
5. It is meaningful to the teachers.

Wittmann concludes that what matters most in teacher preparation programmes is the mathematical component, for it provides students with a comprehensive view of the area of mathematics, which, in turn, enables them to act effectively and purposefully within the field of their expertise.

The silver-tongued narration of J. Jernajczyk and B. Skowron (2018), to be found in their common chapter entitled *Circle and sphere – geometrical speculation* *in philosophy*, vividly links mathematics (the discipline) and Arts. The Authors provide the reader with s review of several metaphors of the circle and sphere, aptly submerged into the gravy of philosophical narration. According to the Authors' declaration, one of the aims of this chapter is to show that "art, hand in hand with mathematics and philosophy becomes a cognitive tool – it constitutes not only an aesthetic complementation of scientific cognition, but it is its reinforcement, deepening and extension" (Jernajczyk, Skowron, 2018, p. 380).

All of the metaphors, being the foci of the chapter, arise from the geometrical properties of the circle and sphere, formulated by way of a multidimensional and profound analysis of these mathematical objects.

The ontological metaphors referred to in this chapter include Parmenides' spherical vision of reality, Plato's roundness of the world, and the geometrical speculations of Nicolas of Cusa. Among the epistemological metaphors recalled by the Authors, one can find scientific knowledge and a dynamic circle collated, the approximation of an ideal circle in metaphors regarding the pursuit of truth and knowledge, as well as Kazimierz Twardowski's metaphor of the research circle.

The present work of Jernajczyk and Skowron is undoubtedly thought-provoking. It draws our attention to some eye-opening features of the circle and sphere. Reflecting on them, we invite the reader to recall and take into account the following famous words stated by R. D. Laing: "If I don't know I don't know, I think I know". They remind us that there still may be some meanings hidden within mathematical objects that we are not aware of, even though we believe we know the objects themselves. In order to explore the unboundedness of mathematics and make the invisible visible to us, we need to call into question our current state of knowledge and transgress the limitations of our current perceptions.

In her paper entitled *Mathematically gifted children: research results, analysis, conclusions,* E. Gruszczyk-Kolczyńska (2018) presents research that focuses on recognising the mathematical aptitude of younger pupils - from the kindergarten stage of learning to their further accomplishments in early primary school education.

The author of the paper, in her own empirical research "Children's mathematics" on older kindergarten children, observed that more than 50% of the children were doing well in mathematics at the early stages of primary school:

> "These children solved math problems with pleasure, using astonishing maths skills. They also discerned maths problems while e.g. doing housework, taking a walk, doing shopping. Their thoughts concerned numbers and measurement. They wanted to measure, calculate, determine proportions, etc. Such an orientation of the mind was determined by Krutetsky to be one of the more important indicators of an aptitude in mathematics" (Gruszczyk--Kolczyńska, 2018, p. 237).

Considering the results of her research, the author formulated a hypothesis which states: "more than half of the overall number of children manifest their mathematics aptitude, as long as proper conditions are maintained at kindergarten and at school" (Gruszczyk-Kolczyńska, 2018, p. 237). In order to verify her hypothesis, the author carried out a research project finished in 2010 named *"Recognising and encouraging the development of aptitude for learning mathemat-*

ics in older kindergartners and primary school students." Referring to the model of mathematics aptitude in older learners by W. A. Krutetsky (1968), she characterised and listed the mental attributes of mathematically gifted kindergarten children:

- The ease of learning maths skills and understanding anything regarding calculating and measuring;
- Achieving the concrete operation stage (in the J. Piaget sense) earlier than others, exhibiting more precise reasoning;
- Easily making sense of situations which require calculating, ordering, determining dependencies, etc. Due to this, they easily perform all the tasks necessary to achieve their goal, spotting errors and properly reacting to absurdities;
- Focusing their attention on sentences for longer periods of time without showing fatigue. Although they will stop upon noticing any sign of ignoring their efforts;
- Doing more attempts at solving tasks if previous attempts were deemed ineffective;
- A creative mindset regarding maths activity. The children look for situations in which they need to make use of calculating, measuring, and organising their environment on their own" (Gruszczyk-Kolczyńska, 2018, pp. 238–239).

In order to carry out her research, the author created her own diagnostic instruments. She delineated 13 areas connected with mathematical activities such as: classification, counting, measuring, geometric intuition, and others. For every area, the author constructed a series of diagnostic tasks at different difficulty levels. As a result of the tests, which were carried out on a total of 487 kindergartners and early primary school students, three groups of children of different mathematical competence were identified:

- *Children with less knowledge and abilities than their peers*
- *Children with varying competence and children mathematically gifted*
- *Mathematically highly gifted children.*

The second part of the article broadly develops some crucial issues concerning educating mathematically gifted pupils and the role of teachers in this field. Some questions arise:

- Why are teachers often wrong in their assessment of the mathematically gifted pupils' mental abilities?
- What are the symptoms of mathematical aptitude in older kindergarten children and early primary school pupils?
- What has already been done to ensure better conditions for mathematically gifted children at the early stage of school education?

To sum up, it can be stated that the research results have shown that young, mathematically gifted children show a startling intellectual curiosity as they keenly participate in activities and games involving great mental effort and combinatorical reasoning. They are able to concentrate for a substantially long time on complex tasks, and, moreover, they can invent such tasks themselves, presenting extraordinary ingenuity. Therefore, it should be emphasised that mathematically gifted children should be given professional support as early as during kindergarten and the earliest years of school education.

References

Błaszczyk, P., Pieronkiewicz, B. (red.): 2018, *Mathematical Transgressions 2015*, Universitas, Kraków.

Cockcroft, W. H.: 1982, *Mathematics Counts (Report of the Committee of Inquiry on the Teaching of Mathematics)*, Her Majesty's Stationery Office, London.

Ernest, P.: 1998, *Social constructivism as a philosophy of mathematics*, State University of New York Press, Albany, New York.

Ernest, P.: 2007, Values and the social responsibility of mathematics, *Philosophy of Mathematics Education Journal* **22**.

Ernest, P.: 2015, Values and mathematics: Overt and covert, *Culture and dialogue* **4**(1), 48–82.

Ernest, P.: 2018, Challenging three myths about mathematics: Recognising the social responsibility of mathematics, w: P. Błaszczyk, B. Pieronkiewicz (red.), *Mathematical Transgressions 2015*, Univeristas, Kraków, 11–50.

Franklin, J.: 2017, Discrete and continuous: a fundamental dichotomy in mathematics, *Journal of Humanistic Mathematics* **7**(2), 355–378.

Gruszczyk-Kolczyńska, E.: 2018, Mathematically gifted children: research results, analysis, conclusions, w: P. Błaszczyk, B. Pieronkiewicz (red.), *Mathematical Transgressions 2015*, Univeristas, Kraków.

Gruszczyk-Kolczyńska, E. (red.): 2012, *O dzieciach matematycznie uzdolnionych. Książka dla rodziców i nauczycieli*, Wydawnictwo Nowa Era, Warszawa.

Hoyles, C., Jones, K.: 1998, Proof in dynamic geometry contexts, w: C. Mammana, V. Villani (red.), *Perspectives on the Teaching of Geometry for the 21st Century*, Dordrecht: Kluwer, 121–128.

Jernajczyk, J., Skowron, B.: 2018, Circle and sphere - geometrical speculations in philosophy, w: P. Błaszczyk, B. Pieronkiewicz (red.), *Mathematical Transgressions 2015*, Univeristas, Kraków, 379–396.

Krauthausen, G., Scherer, P.: 2010, Natural Differentiation in Mathematics (NaDi-Ma). Theoretical Backgrounds And Selected Arithmetical Learning Environments, w: B. Maj, E. Swoboda, K. Tatsis (red.), *Motivation via natural differentiation in mathematics*, Rzeszów, 11–37.

Krutetsky, W. A.: 1968, *Psikhologiia matematicheskikh sposobnostei shkol'nikov*, Prosveshchenie, Moscow.

Mason, J., Graham, A., Johnson-Wilder, S.: 2005, *Developing thinking in algebra*, Sage, London, UK.

Pieronkiewicz, B., Szczygieł, M.: 2017, Transgressing personal foregrounds through the learning of mathematics, *Early Child Development and Care* 1–15.

Wittmann, E. C.: 1995, Mathematics education as a 'design science', *Educational studies in Mathematics* **29**(4), 355–374.

Wittmann, E. C.: 2001, Developing mathematics education in a systemic process, *Educational Studies in Mathematics* **48**(1), 1–20.

Wittmann, E. C.: 2005, Mathematics as the science of patterns – a guideline for developing mathematics education from early childhood to adulthood , *Plenary Lecture at International Colloquium'Mathematical learning from Early Childhood to Adulthood*, (Belgium, Mons).

Wittmann, E. C.: 2018, Structure-genetic Didactical Analyses. Empirical Research "of the first kind", w: P. Błaszczyk, B. Pieronkiewicz (red.), *Mathematical Transgressions 2015*, Univeristas, Kraków, 133–150.

Instytut Matematyki
Uniwersytet Pedagogiczny
ul. Podchorążych 2
PL-30-084 Kraków
e-mail bozena.rozek@up.krakow.pl